6.5: Applications of Common Logarithms

Common Logarithm – the base-10 logarithm, $\log_{10} x$, usually written as $\log x$.

Recall One-to-One Property: If $\log_b x = \log_b y$, then $x = y$. (Converse also true.)

If x & y are positive real #'s and $x = y$, then $\log x = \log y$. ***Common log, both base 10.**

The Common Logarithm can be used to solve exponential equations…

$5^x = 62$

$\log 5^x = \log 62$ *Take common logarithm*

$x \log 5 = \log 62$ *Power property*

$x = \dfrac{\log 62}{\log 5}$ *Division Property*

$x = 2.56$ *Use LOG key on calculator*

Exponential equations can also be solved by graphing…

*Enter on side of equation in y_1, the other side in y_2, and find intersection point.

Solve $5^x = 62$ by graphing.

1) Enter 5^x in y_1 and 62 in y_2.
2) Adjust window so you can see the intersection point.
3) Find intersection point: 2nd Trace, #5.

Solve each equation. Round to the nearest hundredth.

1) $6^x = 82$

$\log 6^x = \log 82$

$\dfrac{x \log 6}{\log 6} = \dfrac{\log 82}{\log 6}$

$x = 2.46$

2) $8^x = 792$

$\log 8^x = \log 792$

$\dfrac{x \log 8}{\log 8} = \dfrac{\log 792}{\log 8}$

$x = 3.21$

3) $3^{-x} = 0.02$

$\log 3^{-x} = \log .02$

$\dfrac{-x \log 3}{\log 3} = \dfrac{\log .02}{\log 3}$

$\dfrac{-x}{-1} = \dfrac{-3.56}{-1}$

$x = 3.56$

4) $8.2^{x+1} = 55$

$\log 8.2^{x+1} = \log 55$

$\dfrac{x+1(\log 8.2)}{\log 8.2} = \dfrac{\log 55}{\log 8.2}$

$x + 1 = \dfrac{\log 55}{\log 8.2}$

$-1 \quad -1$

$x = .90$

5) $40 - 3^x = 29$

$-40 \quad -40$

$\dfrac{-3^x}{-1} = \dfrac{-11}{-1}$

$3^x = 11$

$\log 3^x = \log 11$

$\dfrac{x \log 3}{\log 3} = \dfrac{\log 11}{\log 3}$

$x = 2.18$

6) $5 + 4^{-x} = 15$

$-5 \quad -5$

$\log 4^{-x} = \log 10$

$-x \log 4 = \log 10$

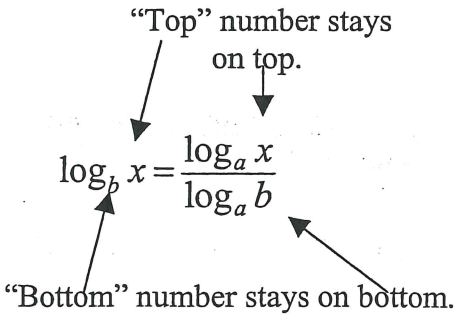
$\dfrac{-x \log 4}{\log 4} = \dfrac{1}{\log 4}$

$\dfrac{-x}{-1} = \dfrac{+1.66}{-1}$

$x = -1.66$

Change of Base Formula

For any positive real number, $a \neq 1, b \neq 1, and \ \ x > 0.$



**Changing a logarithmic expression of any base to base 10 allows us to use the *LOG* button on calculator.

Example: $\log_7 56 = \frac{\log 56}{\log 7} = \frac{1.748}{0.845} = 2.07$

Evaluate each logarithmic expression.

1) $\log_8 97$

$\frac{\log 97}{\log 8}$

2.20

2) $\log_6 3$

$\frac{\log 3}{\log 6}$

.61

3) $\log_{\frac{1}{4}} 16$

$\frac{\log 16}{\log \frac{1}{4}}$

-2

4) $\log_5 \frac{2}{13}$

$\frac{\log \frac{2}{13}}{\log 5}$

-1.16

5) $2 - \log_5 21$

$2 - \frac{\log 21}{\log 5}$

.11

6) $\log_{\frac{1}{4}} 9 + 5$

$\frac{\log 9}{\log \frac{1}{4}} + 5$

3.42